rather than the Rayleigh–Ritz method with empty cavity basis functions. Although this paper refers to the specific boundary problem, conclusions are general and can be useful for other similar cases.

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Propagation in Longitudinally Magnetized Compressible Plasma Between Two Parallel Planes

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Abstract—**The propagation of plasma waves in compressible, single fluid, macroscopic plasma, between two parallel, perfectly conducting planes, with longitudinal magnetostatic field parallel to the boundaries and in the direction of propagation is investigated for the different hybrid plasma wave modes of propagation.**

I. Propagation in Parallel Plane Waveguide

The propagation of plasma waves in compressible, single fluid, macroscopic plasma, between two parallel perfectly conducting planes, with transverse magnetostatic field parallel to the boundaries, has been recently investigated [1]. In the present short paper the theory will be extended to the case where the magnetostatic field is parallel to the boundaries and in the longitudinal direction of propagation of the waves.

Using small signal theory approximation, and assuming harmonic time variation $e^{+i\omega t}$, the wave equation for the electric field $\bar{E}$ in the magnetoplasma has been found [1] in the form

$$-\frac{1}{k_0^2}\nabla\times\nabla\times\bar{E}+\frac{1}{k_1^2}\nabla(\nabla\cdot\bar{E})+(1-X)\bar{E}+\frac{1}{k_0^2}(\nabla\times\nabla\times\bar{E}-k_0^2\bar{E})\times i\bar{Y}=0 \quad (1)$$

where k_0 is the electromagnetic wave number, k_1 is the acoustic wave number, X is proportional to the average plasma density N_0, and $\bar{Y}$ is proportional to the magnetostatic field $\bar{H}_0$. The wave magnetic field $\bar{H}$ and the wave velocity field $\bar{u}$ may be found [1] from the plasma wave electric field $\bar{E}$.

It is assumed that the compressible plasma is confined by two perfectly conducting parallel planes at $x=0$ and $x=a$, with the magnetostatic field in the longitudinal direction of propagation z

$$\bar{Y}=Y\hat{z}=\frac{e\mu H_0}{m\omega}\hat{z}. \quad (2)$$

Manuscript received January 20, 1982; revised September 15, 1982.
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Since the solution will be independent of the y-axis, one may assume that each one of the plasma wave components will be in the form

$$E_j(x,z)=E^j(\alpha)e^{i\alpha x}e^{i(\omega t-\gamma z)}, \qquad j=x,y,z. \quad (3)$$

The constant γ represents the propagation constant of the plasma wave modes propagating in the z direction, and it depends on α to be determined from the boundary conditions.

Substituting (2) and (3) in (1), and taking from (3) $\partial/\partial x=i\alpha$, $\partial/\partial y=0$, $\partial/\partial z=-i\gamma$, one obtains three homogeneous linear algebraic equations for E^x, E^y, and E^z. For a nontrivial solution, the determinant of the coefficients should be zero, and developing this determinant, one obtains

$$\left[k_0^2(1-X)-(\alpha^2+\gamma^2)\right]^2\left[k_0^2(1-X)-\delta(\alpha^2+\gamma^2)\right]+Y^2(k_0^2-\alpha^2-\gamma^2)\left[k_0^2X(k_0^2-\gamma^2)-(k_0^2-\delta\gamma^2)(k_0^2-\alpha^2-\gamma^2)\right]=0 \quad (4)$$

where

$$k_0^2=\omega^2\mu\epsilon$$

and

$$\delta=k_0^2/k_1^2.$$

Equation (4) could be rearranged to give a cubic equation in terms of α^2, with the coefficients of the equation depending on γ^2.

According to the theory of linear algebraic equations, one may express E^x and E^y in terms of E^z. All the other plasma wave components $\bar{H}$ and $\bar{u}$ of the plasma wave hybrid modes could be expressed in terms of E^z as well, by using the relationships given previously [1]. The following boundary conditions will be applied in the present problem:

$$E_z=0 \quad \text{at } x=0 \text{ and } x=a \quad (5a)$$
$$E_y=0 \quad \text{at } x=0 \text{ and } x=a \quad (5b)$$
$$u_x=0 \quad \text{at } x=0 \text{ and } x=a. \quad (5c)$$

II. The Plasma Waves Hybrid Modes

The equation which relates α^2 with the propagation constant γ of the plasma waves hybrid modes is given in (4). For a given γ, one may solve the cubic equation (4) in order to obtain the corresponding characteristic values $\pm\alpha_1$, $\pm\alpha_2$, and $\pm\alpha_3$ in terms of γ. It may be assumed, therefore, that the longitudinal electric field component E_z of the plasma waves hybrid mode is given in the form

$$E_z=[A_1\sin\alpha_1x+B_1\cos\alpha_1x+A_2\sin\alpha_2x+B_2\cos\alpha_2x+A_3\sin\alpha_3x+B_3\cos\alpha_3x]e^{i(\omega t-\gamma z)} \quad (6)$$

where A_1, A_2, A_3 and B_1, B_2, B_3 are arbitrary constants. Using (6) and the analysis described above, one may find E_y in terms of the trigonometric functions and the arbitrary constants in (6) and the constants $D_m=D_m(\alpha_m^2,\gamma)$, where $m=1,2,3$. Using (6) and the corresponding relationship in the previous paper [1], one may find u_x in terms of the trigonometric functions and the arbitrary constants in (6) and the constants $P_m(\alpha_m^2,\gamma)$, where $m=1,2,3$.

Using (6) in the boundary conditions (5a) one obtains

$$B_1+B_2+B_3=0 \quad (7a)$$

$$A_1\sin\alpha_1a+B_1\cos\alpha_1a+A_2\sin\alpha_2a+B_2\cos\alpha_2a+A_3\sin\alpha_3a+B_3\cos\alpha_3a=0. \quad (7b)$$

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Using the expression found for E_y corresponding to (6) in the boundary conditions (5b) one obtains

$$\alpha_1 D_1 A_1 + \alpha_2 D_2 A_2 + \alpha_3 D_3 A_3 = 0 \quad (8a)$$

$$\alpha_1 D_1 A_1 \cos\alpha_1 a - \alpha_1 D_1 B_1 \sin\alpha_1 a + \alpha_2 D_2 A_2 \cos\alpha_2 a - \alpha_2 D_2 B_2 \sin\alpha_2 a + \alpha_3 D_3 A_3 \cos\alpha_3 a - \alpha_3 D_3 B_3 \sin\alpha_3 a = 0. \quad (8b)$$

Using the expression found for u_x corresponding to (6) in the boundary conditions (5c) one obtains

$$A_1 P_1 + A_2 P_2 + A_3 P_3 = 0 \quad (9a)$$

$$A_1 P_1 \cos\alpha_1 a - B_1 P_1 \sin\alpha_1 a + A_2 P_2 \cos\alpha_2 a - B_2 P_2 \sin\alpha_2 a + A_3 P_3 \cos\alpha_3 a - B_3 P_3 \sin\alpha_3 a = 0. \quad (9b)$$

Equations (7), (8), and (9) represent six linear homogeneous equations with the six unknowns A_1, A_2, A_3 and B_1, B_2, B_3, and for a nontrivial solution the determinant of the coefficients should be zero.

Substituting the values of $\alpha_1(\gamma)$, $\alpha_2(\gamma)$, and $\alpha_3(\gamma)$ found from (4) in the above determinantal equation, one obtains a transcendental determinantal equation for the propagation constant γ of the plasma waves hybrid modes. The solution of this equation will give an infinite number of discrete solutions for γ. For each γ of a particular hybrid mode, one may find the corresponding characteristic values α_1, α_2, and α_3, from which one is able to find the field components of the corresponding plasma wave hybrid mode.

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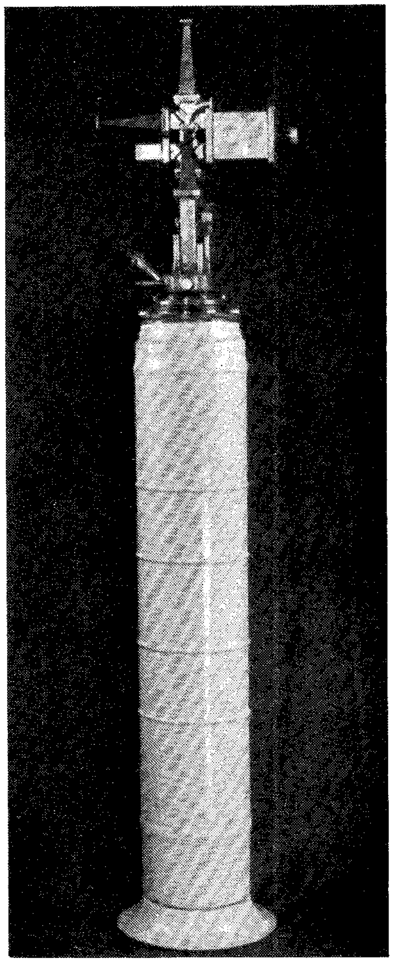

Fig. 1. General view of the maser.

A Broad-Band Traveling-Wave Maser for the Range 40–46.5 GHz

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***Abstract* — A tunable traveling-wave maser (TWM) for the frequency range 40–46.5 GHz has been developed, which is characterized by an extended instantaneous bandwidth. Andalusite (Al_2SiO_5) doped with Fe^3 atoms is used as the active crystal. The slow-wave structure is a digit comb with broad-band matching particularly suitable for the millimeter range. The new type of isolator employed is based on textured hexagonal ferrite materials, namely $BaNi_2Sc_xFe_{16-x}O_{27}$. The net gain within the tuning band is 20–35 dB. The instantaneous bandwidth at a −3-dB level is 150–100 MHz, depending on the net gain. The noise temperature at the input does not exceed 25° K.**

I. Introduction

Making use of the results obtained earlier in the analyses of millimeter-band active crystals [1], the slow-wave structure [2], and ferrites [3], a traveling-wave maser (TWM) has been developed for the frequency range 40 to 46.5 GHz, which is characterized by a high value of the gain and a broad band of amplified frequencies. The preliminary results on this amplifier reported earlier in [4], [5] concerned mainly the higher frequency part of the above frequency range. The present paper contains new experimental results obtained in further investigations, particularly on measurements of the amplifier performance in the SWS passband.

Manuscript received March 31, 1982; revised October 8, 1982. The authors are with the Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian S.S.R., Kharkov, U.S.S.R.

The amplifier employs a number of novel elements, such as andalusite (Al_2SiO_5) with Fe^{3+} ions as an active crystal, the Ni_2W hexaferrite for an isolator, and a comb-type slow-wave structure with smooth transitions to waveguides and other functional elements.

The general appearance of the maser is shown in Fig. 1.

II. Active Crystal, Slow-Wave Structure, and Isolator

The maser employs a natural Fe^{3+} containing crystal of andalusite operating in a magnetic field B_0 oriented at 90° to the z axes of both magnetic complexes of the crystal. The axis z_1 of one complex is along the SWS while z_2 of the other is at 59° to z_2, both axes being perpendicular to B_0. The transition 1–2 is employed as a signal transition. One could pump the transitions 1–3 or 1–4 (see Fig. 2) which are but slightly different in frequency (1–2 GHz). The concentration of Fe^{3+} ions in the crystal is ~ 0.07 percent. The EPR bandwidth at the signal frequency is $\Delta f_L = 150$ MHz, and at the pumping frequency

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